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THE SPECIFICATION OF THE DISTURBANCE FOR EFFICIENT ESTIMATION

BY

ROBERT F. ENGLE

NUMBER 76 AUGUST 1971

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I. Introduction

It was early recognized that estimating the model

1) $y = X\beta + \varepsilon$
$E\varepsilon = 0$
$E\varepsilon\varepsilon' = \sigma^2\Omega$

by OLS was generally inefficient unless $\Omega = I$. If Ω is known a priori, then Aitken's estimator is easily computed and enjoys the best linear unbiased property. In general, however, Ω is not known and Aitken's estimator is not estimable from the data since there are $\frac{(T+1)T}{2}$ additional parameters to estimate and only T observations. Several approximations to Aitken's estimator are possible. One is to make a guess as to the correct covariance structure while the other is to assume that there are only a few parameters in Ω and to estimate them from the data. The latter approach appears most reasonable and has been widely used by econometricians without any careful study of the rationale for particular parameterizations or the consequences of a bad choice. One exception to this void is a paper by Wallis [7] in which he chooses a fourth order autoregressive process to describe the disturbance in a quarterly model since seasonal terms must surely be included in an economic model. Other than Wallis' paper most authors have merely chosen a first order

* The author is indebted to Robert Hall, M.I.T., for helpful conversations on this material.

markov process to approximate the disturbance on the implicit assumption that some adjustment is better than none. The purpose of this paper is to examine that assumption. In Section II, we reproduce some broad efficiency bounds developed by Watson. In Section III, we seek finer bounds for this specific comparison and apply these to the first order markov model in a true second order world. In Section IV, we investigate numerically, the typical efficiencies under various assumptions and in Section V, we present the extracted conclusions.

II. The Watson Bounds

The ordinary least squares (OLS) estimator of model (1) has variance covariance matrix

$$2) \qquad \text{Var } b_{OLS} = (X'X)^{-1} X'\Omega X(X'X)^{-1} .$$

If however a covariance matrix H has been assumed, then the estimator will be

$$3) \qquad b_H = (X'H^{-1}X)^{-1} X'H^{-1}Y$$

which has variance covariance matrix:

$$4) \qquad \text{var } b_H = (X'H^{-1}X)^{-1} X'H^{-1}\Omega H^{-1}X(X'H^{-1}X)^{-1} .$$

It is now our intention to compare (2) with (4) for various choices of H, Ω and X.

The matrices H and Ω have the fortunate property that since they result from covariance stationary processes, their eigenvectors are approximately

identical with eigenvalues which are the values of the spectral density functions evaluated at multiples of $2\pi/T$. The approximation is exact either if the processes are defined as circulants (See Hall [5]) or as the sample size becomes large. Specifically, we can follow Fishman [1, p. 212] constructing a unitary matrix U, where

5) $$[U]_{kj} = \frac{1}{\sqrt{T}} \exp\left(\frac{i\pi}{n}(k-n)(j-n)\right) \quad n\varepsilon\,[\,T/2\,]$$

which, for only one exogenous variable, reduces (2) and (4) in the limit to

6) $$\frac{1}{T}\,\text{var}\; b_{OLS} = \left(\frac{1}{\pi}\int_{-\pi}^{\pi} f_x(\theta)d\theta\right)^{-2} \frac{1}{\pi}\int_{-\pi}^{\pi} f_x(\theta) f_\varepsilon(\theta)d\theta$$

7) $$\frac{1}{T}\,\text{var}\; b_H = \left(\frac{1}{\pi}\int_{-\pi}^{\pi} f_x(\theta) f_H^{-1}(\theta)d\theta\right)^{-2} \frac{1}{\pi}\int_{-\pi}^{\pi} f_x(\theta) f_\varepsilon(\theta) f_h^{-2}(\theta)d\theta$$

respectively since

$$U\Omega U^{*} = \text{Diag}\; f_\varepsilon(\theta_j)$$

$$(UX)^{*}_j (UX)_j = f_x(\theta_j)$$

where $\theta_j = \frac{\pi(j-n)}{n}$ $\quad j = 1, \ldots T$, * means conjugate transpose, and $f_z(\theta_j)$ is the spectral density of the process z at frequency θ_j. Asymptotically the sums are replaced by integrals. From a similar argument Aitken's estimator has asymptotic variance

8) $$\frac{1}{T}\,\text{Var}\; b_\Omega = \left(\frac{1}{\pi}\int_{-\pi}^{\pi} f_x(\theta)\; f_\varepsilon^{-1}(\theta)d\theta\right)^{-1}$$

Thus the asymptotic efficiency of b_H relative to generalized least square (GLS) is

$$9) \quad \mathrm{Eff}_{\Omega}(b_H) = \frac{\left(\int_{-\pi}^{\pi} f_x(\theta)\, f_H^{-1}(\theta)\, d\theta\right)^2}{\int_{-\pi}^{\pi} f_x(\theta_j) f_\varepsilon^{-1}(\theta)\, d\theta \int_{-\pi}^{\pi} f_x(\theta) f_\varepsilon(\theta) f_H^{-2}(\theta)\, d\theta} .$$

The subscript on Eff will be taken to be the alternative estimator which in this case is GLS where the covariance matrix is assumed to be Ω. This expression has an upper bound of 1 by Jensen's inequality and from Watson and Hannan [9] and Watson [8] we find the lower bound to be

$$10) \quad \mathrm{Eff}_{\Omega}(b_H) \geq 4 \frac{\left(\frac{f_\varepsilon(\theta_k)}{f_H(\theta_k)}\right)\left(\frac{f_\varepsilon(\theta_1)}{f_H(\theta_1)}\right)}{\left(\frac{f_\varepsilon(\theta_k)}{f_H(\theta_k)} + \frac{f_\varepsilon(\theta_1)}{f_H(\theta_1)}\right)^2}$$

where θ_k and θ_1 are the frequencies where f_ε/f_H reaches its minimum and maximum. Clearly if this ratio is a constant, then the lower bound is unity and Eff = 1. The lower bound is taken on when $\frac{f_x(\theta)}{f_H(\theta)}$ is non-zero at only two frequencies θ_k and θ_1 and at these frequencies, f_x/f_H is the same. The upper bound is realized if f_e/f_H is constant everywhere that f_x is non-zero. Thus, for a given Ω and H, the actual efficiency depends on the spectrum of the exogenous variable.

The relative efficiency of assuming H as opposed to OLS is then

given by

$$11) \qquad Eff_{OLS}(b_H) = \frac{Eff_{\Omega}(b_H)}{Eff_{\Omega}(b_{OLS})} \quad .$$

Letting L_H and L_{OLS} be the Watson lower bounds for the efficiency of each of these estimators, we obtain the broad bounds

$$12) \qquad L_H < Eff_{OLS}(b_H) < \frac{1}{L_{OLS}} \quad .$$

Since our choice of estimator must rest on whether this term is less than or greater than one, this inequality is not particularly helpful. In particular, it is very unlikely that there exists an X process which would make either of these bounds realizable. From (1) we see that if f_ϵ is close to zero for any frequency, L_{OLS} will be very small and if it does so in a way that f_H cannot reproduce, then so will L_H. Watson has calculated the values of these lower bounds for several situations and they are remarkable for being often close to zero. Clearly, a bad choice of H can make L_H arbitrarily close to zero.

III. Efficiency of the First Order Markov Model

In order to narrow the bounds on H relative to OLS we will prove two lemmas. First let us abbreviate $\dfrac{\int_o^\pi f_x(\theta) f_z(\theta) d\theta}{\int_o^\pi f_x(\theta) d\theta}$ by $E[f_z(\theta)]$ where it must be remembered that there is no random component; this is merely notation. Then we can write the basic efficiency definition as

$$13) \qquad Eff_{OLS}(b_H) = \frac{Ef_\epsilon(\theta) E^2 f_H^{-1}(\theta)}{Ef_\epsilon(\theta) f_H^{-2}(\theta)}$$

Lemma 1: A necessary condition for the lower bound of (13) to be greater than unity is that $f_\varepsilon(\theta)$ and $f_H^{-1}(\theta)$ be negatively correlated for all $f_x(\theta)$. That is, $Ef_\varepsilon(\theta)f_H^{-1}(\theta) - Ef_\varepsilon(\theta)Ef_H^{-1}(\theta) < 0$ for any $f_x(\theta)$.

Proof: Suppose there exists an $f_x(\theta)$ such that $Ef_\varepsilon(\theta)f_H^{-1}(\theta) \geq Ef_\varepsilon(\theta)Ef_H^{-1}(\theta)$. Because $f_\varepsilon(\theta) > 0$,

$$E[f_\varepsilon(\theta)(f_H^{-1}(\theta) - Ef_H^{-1}(\theta))^2] > 0$$

unless $f_H^{-1}(\theta)$ is constant, in which case, H is identical with OLS. Squaring, we have

$$Ef_\varepsilon(\theta)\, f_H^{-2}(\theta) - 2E[f_\varepsilon(\theta)f_H^{-1}(\theta)]Ef_H^{-1}(\theta) + Ef_\varepsilon(\theta)E^2f_H^{-1}(\theta) > 0$$

and by hypothesis the middle terms can be replaced, giving

$$Ef_\varepsilon(\theta)f_H^{-2}(\theta) - Ef_\varepsilon(\theta)\, E^2f_H^{-1}(\theta) > 0$$

which shows that for this $f_x(\theta)$, OLS is superior to H.

Lemma 2: For continuous functions $u(\theta)$, $v(\theta)$, if there is a point θ_0 where the slopes are the same sign, then there exists a density function $f(\theta)$ which makes these functions positively correlated.

Proof: Let $f = \begin{cases} \alpha & \theta = \theta_1 \\ 1-\alpha & \theta = \theta_2 \\ 0 & \text{otherwise} \end{cases}$

$$Eu(\theta)v(\theta) - Eu(\theta)Ev(\theta) = \alpha(1-\alpha)[u(\theta_1)-u(\theta_2)][v(\theta_1)-v(\theta_2)]$$

Let $N(\theta_0)$ be the neighborhood of θ_0 where the slopes are of the same sign. Then letting $\theta_1, \theta_2 \in N(\theta_0)$ gives the result. By equating $u(\theta)$ with $f_\varepsilon(\theta)$ and $v(\theta)$ with $f_H^{-1}(\theta)$ this lemma with lemma 1 establishes that a necessary conditions for the lower bound to be greater than one is that $f_\varepsilon(\theta)$ and $f_H^{-1}(\theta)$ must have opposite slopes.

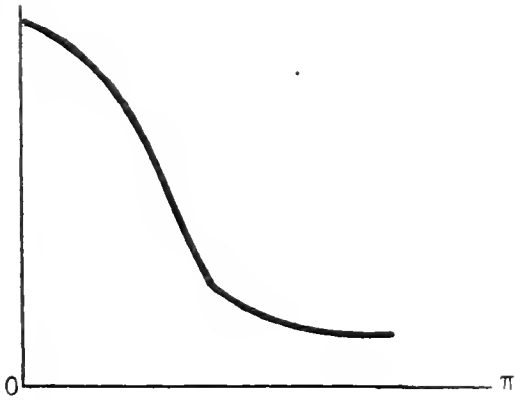
We now apply this result to the assumption that the disturbance is generated by a first order markov process. This assumption means that $f_H(\theta) = \frac{1}{\left|1-\rho e^{i\theta}\right|^2}$ where ρ is the estimated first order serial correlation coefficient of ε which asymptotically is

$$15) \qquad \rho = \frac{E\ \varepsilon\varepsilon_{-1}}{E\ \varepsilon^2} \quad .$$

This estimator will be called autoregressive least squares (ALS) and f_{ALS}^{-1} has the property that it is either always increasing or always decreasing depending on whether ρ is positive or negative respectively. For most general processes, $f_\varepsilon(\theta)$ will change slopes and so ALS is not guaranteed to be an improvement.

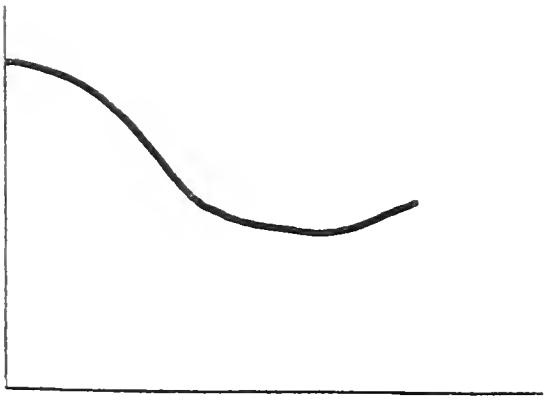
To be more specific, we will assume that the true disturbance is generated by a second order process and investigate the characteristics of ALS. Describing the second order process by its autoregressive roots, γ_1 and γ_2, there are six possible configurations of the power spectrum as shown in figure 1 on the following page. In addition, if one of the roots is very small (roughly less than .1), then some of these situations look instead, just like the first order cases (a) and (e). We will not

(a)



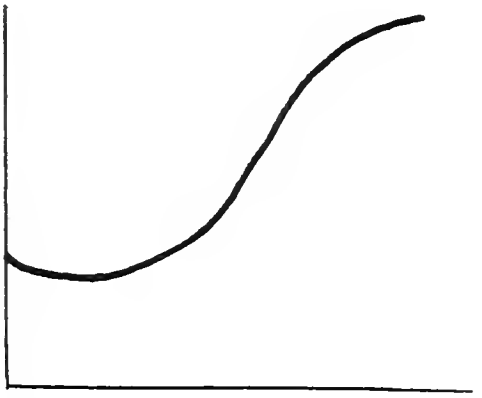
$\gamma_1, \gamma_2 > 0$

(b)



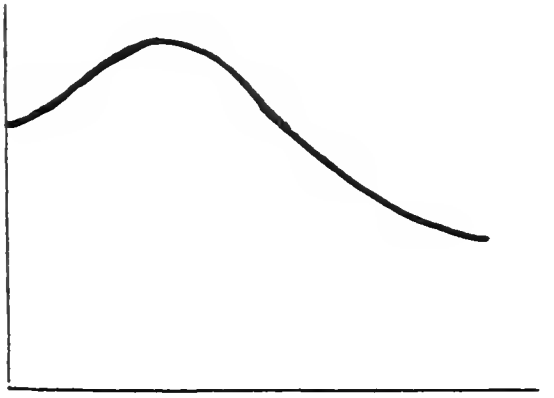
$\gamma_1 > 0, \gamma_2 < 0, \gamma_1 + \gamma_2 > 0$

(c)



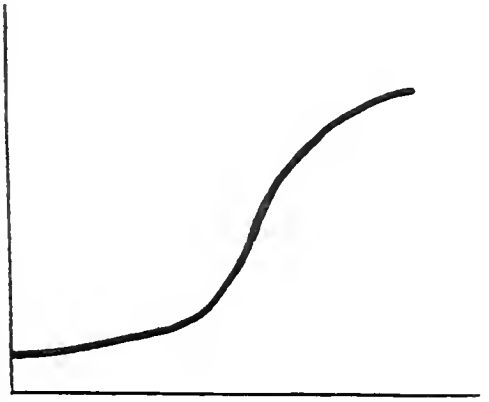
$\gamma_1 > 0, \gamma_2 < 0, \gamma_1 + \gamma_2 < 0$

(d)



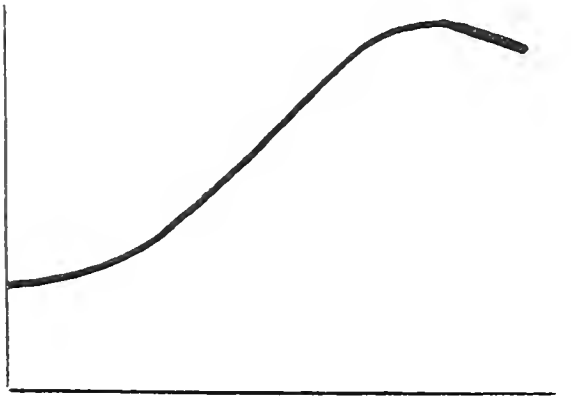
$\gamma_1 = \gamma_2^* \quad Re\gamma < 0$

(e)



$\gamma_1, \gamma_2 < 0$

(f)



$\gamma_1 = \gamma_2^* \quad Re\gamma < 0$

Spectra of second order markov processes with roots γ_1, γ_2.

FIGURE 1

consider these cases as they are clearly so close to first order, that there is litte loss from assuming them to be first order. The estimate of ρ for a second order model is simply $\rho = (\gamma_1+\gamma_2)/(1+\gamma_1\gamma_2)$ which is positive, roughly if the overall slope of the spectrum is negative and negative if it slopes upward.

From lemmas 1 and 2 we see that if the underlying model disturbance is of types (b), (c), (d), or (f), then the efficiency of ALS relative to OLS has a lower bound below one and ALS is not necessarily superior to OLS. Only cases (a) and (e) remain for possible dominance of ALS and from lemma 3, we see that these do satisfy the necessary condition of lemma 1.

Lemma 3: If $u(\theta)$ and $v(\theta)$ are both non-decreasing (increasing) or both non-increasing (decreasing) differentiable functions in the interior of the interval, then they will be non-negatively (positively) correlated for any density function. If they move in opposite directions, the corresponding results will hold.

Proof: $Eu(\theta)v(\theta) - Eu(\theta)Ev(\theta)$

$$= E[(u(\theta) - u(\lambda)\ )\ (v(\theta) - v(\lambda)\ )\ ]$$

where $v(\theta) = v(\lambda)$ and the expectations are taken to be integrals. Apply the mean value theorem

$$= E[u'(\theta^{*})\ v'(\theta^{**})(\theta-\lambda)^{2}]$$

which has all elements greater than or equal to zero since θ^{*} and θ^{**} must always be in the interior of the interval. We have

established that ALS cannot dominate OLS for any second order process except that in (a) or (e) but we have not shown anything about cases (a) and (e).

We next develop a sufficient condition for the lower bound to be greater than unity.

Lemma 4: A necessary and sufficient condition for the lower bound to be greater than unity is that $f_H^{-1}(\theta)$ is negatively correlated with $f_\varepsilon(\theta)(f_H^{-1}(\theta) + Ef_H^{-1}(\theta))$ for all $f_x(\theta)$.

Proof: By hypothesis

$$Ef_\varepsilon(\theta)f_H^{-2}(\theta) + Ef_\varepsilon(\theta)f_H^{-1}(\theta)Ef_H^{-1}(\theta) - Ef_H^{-1}(\theta)[Ef_\varepsilon(\theta)f_H^{-1}(\theta) + Ef_\varepsilon(\theta)Ef_H^{-1}(\theta)] < 0$$

which simplifies to give

$$Ef_\varepsilon(\theta)f_H^{-2}(\theta) - Ef_\varepsilon(\theta)E^2f_H^{-1}(\theta) < 0.$$

If this is true for all $f_x(\theta)$, then the lower bound is greater than one. By lemma 3 we see that we require that $f_H^{-1}(\theta)$ be always increasing and $f_\varepsilon(\theta)\ (f_H^{-1}(\theta) + Ef_H^{-1}(\theta))$ always decreasing or vice versa.

Calculation of $[f_\varepsilon(\theta)(f_H^{-1}(\theta) + Ef_H^{-1}(\theta))]'$ for θ near zero shows that the term has the sign of $-\sin\theta[(\gamma_1 + \gamma_2)(1 + \gamma_1\gamma_2) - \gamma_1\gamma_2]$ while $f_H^{-1'}(\theta)$ has the sign of $\sin\theta(\gamma_1+\gamma_2)/(1+\gamma_1\gamma_2)$.

For both $\gamma_1, \gamma_2 > 0$ and neither particularly small, these signs will be the same. For γ_1 and $\gamma_2 < 0$, the situation will be identical

for $\theta = \pi$. Thus for none of the six types of second order process, is the lower bound greater than unity. Only when one of the roots is very small is it possible that ALS is uniformly superior to OLS.

III. Typical Efficiency Levels

In the face of these pessimistic results for ALS estimation, we must ask what efficiency levels are reasonable in standard econometric practice, rather than whether the lower bound is greater or less than unity. Grenander [3] and Grenander and Rosenblatt [4] established that a general condition for OLS to be fully efficient is that, the spectrum of the disturbance must be flat at all points where the exogenous variable has spectral weight. For economic times series, the exogenous variables typically, (See Granger [2]), have their spectral weight at low frequencies and OLS will be efficient if the disturbance spectrum is flat at low frequencies. Malinvaud [6] suggests as a rule of thumb that only if the first order serial correlation "considerably exceeds" 0.5 should we look for an estimator other than OLS. Consequently, moving from OLS to ALS which is itself an inefficient estimator, should not yield much of a gain in efficiency unless serial correlation is substantial.

In order to explore the typical situation we will examine the efficiency of ALS relative to OLS for exogenous variables which have their entire spectral weight in the lower part of the spectrum. Returning to the classifications in Figure 1, it can

be shown using lemmas 3 and 4 that for processes (b), (e) and (f), ALS will always be superior if the spectral weight of X is entirely in the first half of the diagram. On the other hand, for processes (c) and (d), OLS will dominate if the spectrum of X falls entirely below the point of the minimum in (c) or max in (d) (actually the point at which the division occurs is generally somewhat beyond this critical point). Thus we are left with only case (a) which is uncertain.

We established in the previous section, that for case a, OLS would be superior in the neighborhood of the origin. The percentage of the spectrum over which this condition is valid was computed [1] numerically for several values of the real positive autoregressive parameters and for the complex roots for which the imaginary part is so small that the spectrum has no turning point. These precentages are shown in tables 1 and 2.

---

[1] The value of $Ef_H^{-1}(\theta)$ clearly depends on waht X is being considered. Since the emphasis is on low frequency X's the value of $f_H^{-1}(0)$ was used throughout. This has the effect of slightly extending the range of the condition supporting OLS.

TABLE 1

Percent of spectrum starting at origin
for which OLS is more efficient than ALS.
Real Positive Autoregressive Roots

γ_1=	0.0	0.1	0.3	0.6	0.7	0.8	0.9	0.95	0.98	0.99
γ_2=0.0	0	0	0	0	0	0	0	0	0	0
0.1		0	0	0	0	0	0	0	0	0
0.3			12	16	14	12	8	6	4	2
0.6				14	12	10	7	4	3	2
0.7					10	8	5	4	2	1
0.8						6	4	3	2	1
0.9							3	2	1	1
0.95								1	1	0
0.98									0	0
0.99										0

TABLE 2

Percent of spectrum, starting at origin,
for which OLS is more efficient than ALS.
Complex autoregressive Roots[1]

	Period ∞	20	10	6.667	5	4	3.33	2.22
[γ]= .1	0	0	0	0	21	50	0	0
.2	0	0	0	21	35	50	0	0
.3	12	15	21	29	38	50	0	0
.4	18	20	24	31	39	50	0	0
.5	17	19	24	31	39	50	0	0
.6	14	16	23	30	39	50	0	0
.7	10	14	21	30	39	50	0	0
.8	6	11	20	30	39	50	0	0
.9	3	10	19	29	39	50	0	0
.98	0	9	19	29	39	50	0	0

1/ Cases below dashed line are of type (d), those above are (a). Periods shorter than 4 are symmetrically types (e) and (f).

The conclusions to be drawn from these tables are rather mixed. It is unreasonable to expect that any exogenous variable will have its spectral weight entirely in the first few percent of the spectrum. Thus, for large correlations, we would expect OLS to be inefficient. For intermediate values of the autoregressive parameters, the percentages are higher,suggesting that for plausible series, OLS may definitely be superior. However, as we observed before; when at least one parameter is small, ALS becomes the optimum estimating procedure again.

In order to clear the ambiguity of these results we examined the efficiency of ALS versus OLS for a specific exogenous variable which has the typical spectral shape and which appears to flatten the spectrum of many economic time series. The process was assumed to be second order with both roots equal to .8. The relative efficiencies of the estimators in this environment are presented in tables 3 and 4 for the same cases as in 1 and 2.

TABLE 3

Relative efficiency of ALS vs. OLS when exogenous variable is second order with roots both .8.
Real Positive Autoregressive Roots

	0.0	0.1	0.3	0.6	0.7	0.8	0.9	0.95	0.98	0.99
0.0	1.0	1.0	1.01	1.07	1.17	1.48	2.89	6.58	19.87	54.18
0.1		1.00	1.01	1.08	1.19	1.53	3.02	7.08	21.	58.52
0.3			1.20	1.09	1.22	1.64	3.56	8.39	25.51	69.63
0.6				1.24	1.53	2.36	5.39	12.74	38.68	105.7
0.7					1.95	3.01	6.98	16.40	14.80	136.4
0.8						4.63	10.55	24.63	75.19	207.2
0.9							23.56	55.51	175.6	497.5
0.95								135.9	467.4	1409.
0.98									1949.	6685.
0.99										24508.

TABLE 4

Efficiency of ALS relative to OLS when exogenous variable is second order with both roots .8.
Complex Roots [1]

	Period ∞	20	10	6.667	5	4	3.33	2.22
[γ]=.1	1.002	1.001	1.001	1.000	1.000	1.000	1.000	1.001
.2	1.008	1.007	1.004	1.001	1.000	1.000	1.001	1.004
.3	1.020	1.015	1.006	1.000	.999	1.000	1.002	1.010
.4	1.034	1.018	.997	.993	.996	1.000	1.004	1.019
.5	1.062	.999	.957	.971	.989	1.000	1.007	1.036
.6	1.237	.979	.859	.926	.977	1.000	1.012	1.071
.7	1.953	1.008	.698	.845	.957	1.000	1.019	1.149
.8	4.632	1.027	.494	.714	.920	1.000	1.033	1.343
.9	23.56	.811	.271	.501	.832	1.000	1.073	1.951
.98	1949.	.379	.092	.204	.556	1.000	1.338	6.591

1/ See note to table 2.

These tables give two results. First, for type (a) processes the efficiency never falls below .96 although it rises to several thousand for very high autocorrelations. Secondly, however, for moderate values of the parameters (i.e., first order serial correlation less than .6) the maximum of the efficiency of ALS is 1.07. Thus, while typical values for the efficiency under case (a) suggest that ALS may be preferable, the gain in efficiency may not be worth the effort of using another estimator. This result is radically changed if the values of the parameters indicate pronounced serial correlation, in which case, ALS is far superior to OLS.

In conclusion,for underlying second order processes of type (a), (b), (e) and (f), ALS is the recommended procedure while for types (c) and (d),OLS should generally give a superior estimate. In practice, the characteristics of the true distrubance process could be determined by estimating the equation by OLS and then computing the first order serial correlation coefficient. If its size is significant, spectral density function of the residuals would then be estimated, since asymptotically it is the spectrum of the disturbance, and thus the process would be roughly classified into one of the six types.

IV. Conclusion

We have examined the asymptotic efficiency of estimators which assume a first order markov process for the disturbance relative to ordinary least squares, when the true disturbance is instead a second

order markov process. The efficiency depends on the exogenous variable in the equation and it is shown that only when the second order process has one root near zero, is ALS necessarily more efficient. However, for econometric applications, we can look specifically at the low frequency segment of the spectrum. Separating the types of spectra for the second order process into six general shapes, we can then choose which estimator is likely to be superior. If the spectrum of the disturbance has an overall slope which is opposite to the slope at low frequencies, then OLS should be superior; otherwise, ALS is recommended.

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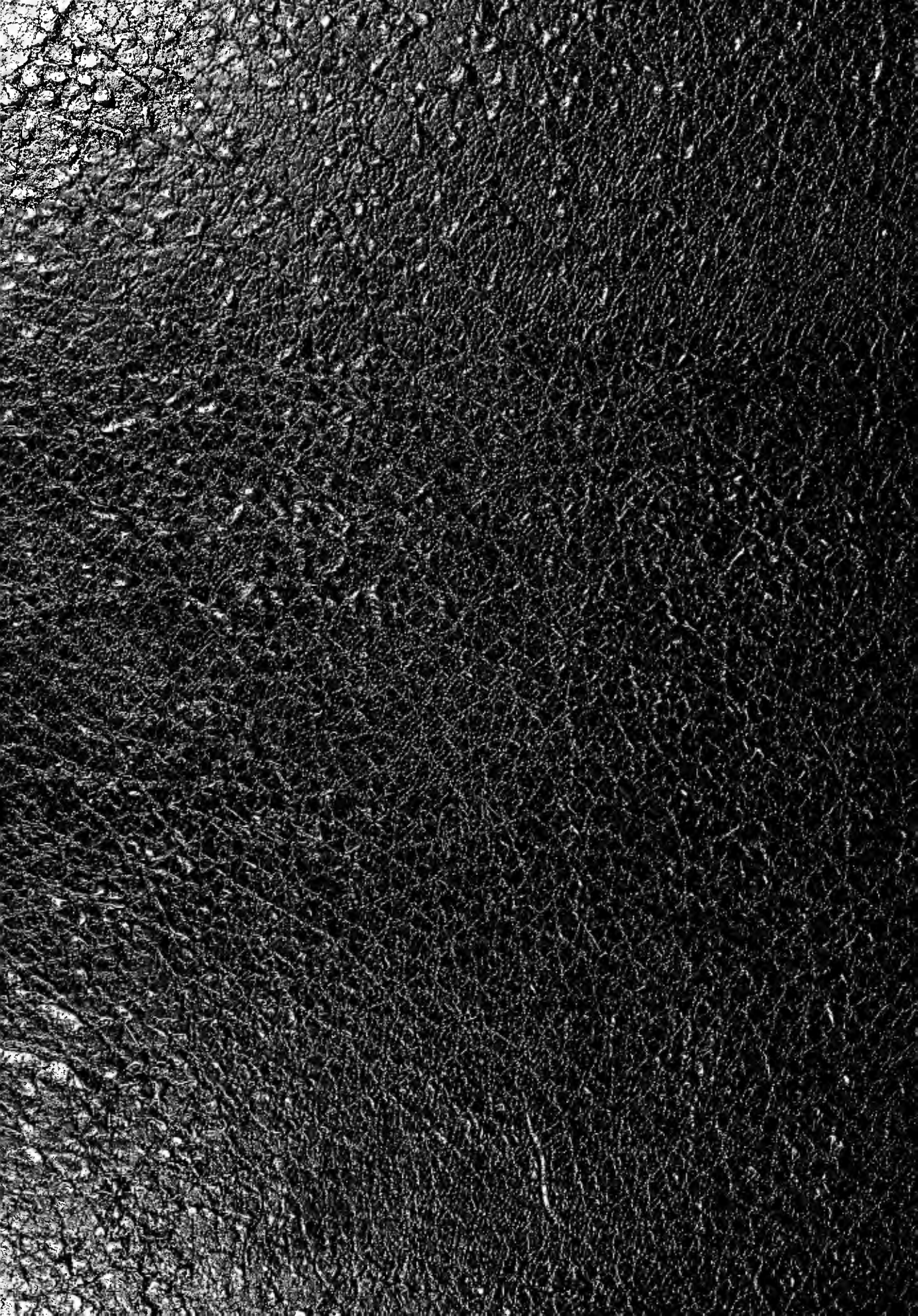
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